pipe radius; L, length of active region of condenser; Re_r, radial Reynolds number; $\tilde{X}$, axial coordinate; $\tilde{Y}$, radial coordinate; Y, dimensionless radial coordinate; B, geometric parameter; ρ, density; ν, kinematic viscosity; ψ, current function; V, dimensionless radial velocity component; X, dimensionaless axial coordinate.

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THEORY OF AN ABSOLUTE SUPERCONDUCTING BOLOMETRIC THERMAL-RADIATION RECEIVER

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Taking the transient zone into account, a theory is developed and the operation is analyzed for a superconducting nonisothermal bolometer in the regime of absolute thermal-radiation reception.

The main problem in producing an absolute thermal-radiation receiver (ATRR) is assurance of the equivalence of the electrical substitution power and the radiant thermal flux power absorbed by the ATRR sensor. One of the promising areas in the solution of this problem is the production of an ATRR based on a superconducting nonisothermal bolometer (SNB), first proposed by Franzen [1]. However, the theory worked out in [1] is developed for the two-phase state of the SNB, i.e., without taking account of the transient zone from the normal to the superconducting state, for the case when the incident thermal flux is distributed uniformly over the whole surface of the ANB sensor. This makes direct utilization of the SNB of known structures [2-4] difficult for the production of an ATRR because of the different nature of the thermal energy absorption and liberation by the bolometer sensor. In this paper a three-phase (taking account of the transient zone) theory is developed for the SNB, and computations are performed for the case when the incident thermal flux is distributed

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symmetrically over a normal section of the SNB sensor, and appropriate corrections are also obtained for the computational formulas of the thermal substitution method.

The distribution of temperature T along the surface of an SNB sensor, whose length and breadth are much greater than the thickness, is described by a one-dimensional heat-conduction equation with the following form in the general case:

$$C\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left(\lambda_э\frac{\partial T}{\partial x}\right)+\frac{W}{2x_0(A_S+A_0)}+\frac{\rho i^2}{A_S(A_S+A_0)}, \tag{1}$$

where x is the distance from the center of the bolometer. In the stationary case, when the heat flux W is distributed symmetrically in the section ($-x_0 \le x \le x_0$) over the sensor surface, by taking account of the transient zone whose boundaries (x_{c1}, x_{c2}) correspond to the transition temperatures (T_{c1}, T_{c2}), we can dissociate (1) into four equations which can be rewritten as follows after reduction to dimensionless form:

$$a_N^2\frac{d^2\Theta}{dy^2}+\frac{\eta}{y_0}+1=0;\ 0\leqslant y\leqslant y_0\,(T_{\max}\geqslant T\geqslant T_0), \tag{2}$$

$$a_N^2\frac{d^2\Theta}{dy^2}+1=0;\ y_0\leqslant y\leqslant y_{c1}\,(T_0\geqslant T\geqslant T_{c1}), \tag{3}$$

$$\frac{d}{dy}\left[(a_N^2+\alpha\Theta)\frac{d\Theta}{dy}\right]+\gamma\Theta+1=0;\ y_{c1}\leqslant y\leqslant y_{c2}\,(T_{c1}\geqslant T\geqslant T_{c2}), \tag{4}$$

$$a_S^2\frac{d^2\Theta}{dy^2}=0;\ y_{c2}\leqslant y\leqslant 1\ (T_{c2}\geqslant T\geqslant T_b). \tag{5}$$

Here $a_e^2=\dfrac{\lambda_e A_S(A_S+A_0)(T_{c1}-T_b)}{\rho L^2 i^2}$ is the dimensionless heat conduction, a current parameter; $\eta = WA_S/2\rho Li^2$ is the dimensionless power absorbed by the bolometer. It is here assumed that the dimensionless effective heat conduction of the bolometer changes linearly in the temperature with the proportionality factor $\alpha=\gamma(a_N^2-a_S^2)$ during the transition from the normal (with the subscript N) to the superconducting (subscript S) state, while a_N^2 and a_S^2 are constants equal to the integrated mean in the temperature range under consideration.

Integration of (2) and (3) is trivial and yields

$$a_N^2\Theta=-\frac{1}{2}y^2\left(\frac{\eta}{y_0}+1\right)+\eta\left(y_{c1}-\frac{1}{2}y_0\right)+\frac{1}{2}y_{c1}^2,\ 0\leqslant y\leqslant y_0, \tag{6}$$

$$a_N^2\Theta=\eta(y_{c1}-y)+\frac{1}{2}(y_{c1}^2-y^2),\quad y_0\leqslant y\leqslant y_{c1}, \tag{7}$$

while we introduce the new variable $d\Theta/dy=P(\Theta)$ for the integration of (4), whereupon (4) goes over into the Bernoulli equation whose solution is well known [5], and after simple algebraic manipulations we obtain

$$\frac{dy}{d\Theta}=-(a_N^2+\alpha\Theta)\left[(\eta+y_{c1})^2-a_N^2\frac{(\gamma\Theta+1)^2-1}{\gamma}-\alpha\Theta^2\left(\frac{2}{3}\gamma\Theta-1\right)\right]^{-1/2}. \tag{8}$$

For a repeated integration of (8) it is necessary to expand the right side in a power series in $(\alpha\Theta)$, after which the integration can be performed analytically with any previously assigned accuracy by giving the quantity of series terms. We limit ourselves in the subsequent consideration to the linear term in the expansion, which corresponds to the condition

$$\alpha/a_N^2\gamma\ll 1, \tag{9}$$

and we then obtain from (8)

$$y = y_{c2} - \left(\frac{a_N^2}{\gamma}\right)^{1/2} \left\{\left(1 - \frac{\alpha}{a_N^2\gamma}\right) \arcsin \frac{a_N(\gamma\Theta + 1)}{[(\eta + y_{c1})^2\gamma + a_N^2]^{1/2}} - \right.$$

$$\left. - \frac{\alpha}{a_N^3\gamma}[(\eta + y_{c1})^2\gamma + a_N^2 - a_N^2(\gamma\Theta + 1)^2]^{1/2} + \frac{\alpha}{a_N^3\gamma}[(\eta + y_{c1})^2\gamma + a_N^2]^{1/2}\right\}, \quad y_{c1} \leqslant y \leqslant y_{c2}. \tag{10}$$

The equation obtained describes the temperature distribution in the transition zone in a linear approximation in α.

Integrating (5) with (8) taken into account yields

$$a_S^2\Theta = (1 - y)\left[(\eta + y_{c1})^2 + \frac{a_N^2}{\gamma}\right]^{1/2} - a_S^2, \quad y_{c2} \leqslant y \leqslant 1, \tag{11}$$

from which we obtain the following equation to determine the limits y_{c1} and y_{c2} of the transition zone by setting $y = y_{c1}$, $\Theta = 0$ in (10) and $y = y_{c2}$, $\Theta = -1/\gamma$ in (11):

$$a_S^2\left(1 - \frac{1}{\gamma}\right) = (1 - y_{c2})\left[(\eta + y_{c1})^2 + \frac{a_N^2}{\gamma}\right]^{1/2}, \tag{12}$$

$$y_{c2} - y_{c1} = \left(\frac{a_N^2}{\gamma}\right)^{1/2} \frac{a_S^2}{a_N^2} \arcsin \frac{a_N}{[(\eta + y_{c1})^2\gamma + a_N^2]^{1/2}} + \frac{\alpha}{a_N^2\gamma}\left[\sqrt{(\eta + y_{c1})^2 + \frac{a_N^2}{\gamma}} - (\eta + y_{c1})\right]. \tag{13}$$

All the fundamental SNB characteristics can be obtained from (6), (7), and (10)-(13); however, their solution in general form is possible only numerically on an electronic computer.

A numerical analysis of the equations obtained showed that for each fixed value of γ, λ_S and λ_N there exists a critical parameter $(a_S^2)_{cr} = i_0^2/i_{cr}^2$, meaning that there also exists a critical current i_{cr} below which $(a_S^2 > (a_S^2)_{cr})$ a nonisothermal bolometer operation regime is not realized. Results of computations executed on the basis of (12)-(13) for $\eta = 0$ and different fixed values of the ratio λ_S/λ_N are represented in Fig. 1, from which it is seen that the critical current drops abruptly as $\gamma < 5$-10 diminishes, while the width of the transition zone, and therefore its influence on the SNB regime, increase abruptly; however, this influence already becomes insignificant for $\gamma \geqslant 20$.

Strictly speaking, (10) and (15) are valid in the case when the effective heat conduction of the bolometer varies weakly during its transition from the normal to the superconducting state, i.e., when $\lambda_S/\lambda_N \gtrsim 0.8$-$0.9$.

For real bolometers deposited on glass [2] or mica [4] substrates, this inequality is always satisfied since the effective heat-conduction coefficient of such bolometers is determined mainly by the heat conduction of the substate for a relationship $A_S/A_0 \leqslant 10^{-2}$ - 10^{-1} between the film and substrate areas. Thus, for a lead bolometer 500 Å thick deposited on a 5-μm-thick mica substrate, the ratio is $\lambda_S/\lambda_N \approx 0.9$, while the heat conduction of the lead during the transition into the superconducting state diminishes fivefold. Moreover, as follows from the computations performed, even in the case of a strong change in the bolometer effective heat conduction in the transition zone ($\lambda_S/\lambda_N = 0.5$), the influence of α on the bolometer parameters is negligible (not more than 5% for i_{cr}, y_{c1}, and y_{c2}) and taking account of the next approximations in α cannot raise the accuracy of the computations substantially, which permits utilization of (13) for a further quantitative analysis. The computations also showed, and this follows directly from (12)-(13), that the width of the transition zone and the zone of the normal state, meaning also the sensitivity of the SNB, are practically independent of the nature of the incident radiant heat flux distribution over the length of a normal section of the bolometer, and depend only on its magnitude, which permits utilization of such a bolometer to produce an absolute thermal-radiation receiver with an independent band sensitivity.

To analyze the influence of the transition zone on the fundamental SNB characteristics, we expand the arcsin z in the right side of (13) in a series and limit ourselves to the first

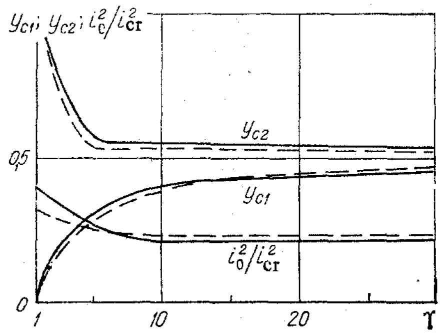

Fig. 1

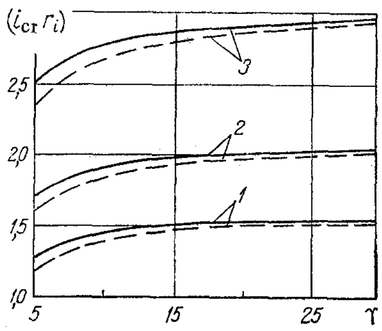

Fig. 2

Fig. 1. Dependence of the critical current and transition zone boundaries on the transition torsion parameter γ for $i = i_{cr}$. Solid curves are for $\lambda_S/\lambda_N = 1$ and the dashes for $\lambda_S/\lambda_N = 0.5$; $\eta = 0$.

Fig. 2. Dependence of the dimensionless bolometer sensitivity in the constant current regime $i_{cr}r_i$ for $\eta = 0$ on the transition torsion parameter γ. Solid curves are for $\lambda_S/\lambda_N = 1$, dashes for $\lambda_S/\lambda_N = 0.5$; and 1, 2, 3) $i_{cr}/i = 0.93, 0.95, 0.97$.

term. Then upon compliance with the condition

$$a_N^2/\gamma \ll 1, \tag{14}$$

i.e., when the difference between the bolometer critical temperatures $\Delta T_c = (T_{c1} - T_{c2})$ is much less than the temperature of bolometer supercooling $\Delta T = T_{c1} - T_b$, which can always be reached by reducing the temperature of the bolometer base, the solution of (12)-(13) can be written in the form

$$y_{c2} - y_{c1} = \frac{1}{2} \frac{a_N^2 + a_S^2}{\gamma(\eta + y_{c1})}, \tag{15}$$

$$y_{c1} = \frac{1}{2} \left\{ (1 - \eta) \pm \sqrt{(1 + \eta)^2 - 4a_S^2 - \frac{2\alpha}{\gamma^2}} \right\}, \tag{16}$$

where the minus sign in front of the radical corresponds to the unstable state of the bolometer in the nonisothermal regime.

It follows from (16) that a critical current exists for $\eta = 0$ determined from the condition

$$D^2(i_{cr}) = 1 - 4a_S^2 - \frac{2\alpha}{\gamma^2} = 0 \tag{17}$$

or in dimensional form

$$i_{cr} = \sqrt{\left(1 + \frac{\lambda_N - \lambda_S}{2\gamma\lambda_S}\right) \frac{4\lambda_S A_S (A_S + A_0)(T_{c1} - T_b)}{\rho L^2}}, \tag{18}$$

for values below which a stable SNB state is impossible; the minimal half-length of the normal section cannot here be less than the critical value $x_{cr} = L/2$.

To determine the SNB sensitivity, the total bolometer resistance must be calculated with the transition zone taken into account, whereupon we write it in the dimensionless form

$$Y_b = 2y_{c1} + 2 \int_{y_{c1}}^{y_{c2}} (\gamma\Theta + 1)\, dy, \tag{19}$$

from which we finally obtain

$$Y_b = 2y_{c1} + \frac{1}{2}\frac{a_N^2 + a_S^2}{\gamma(\eta + y_{c1})} \tag{20}$$

by using (10) and (15) and taking account of (14). To obtain the SNB sensitivity in the constant current regime

$$r_i = \frac{1}{i}\left(\frac{\partial Y_b}{\partial \eta}\right)_i \tag{21}$$

we expand the right side of (16) in a power series in η and limits ourselves to the linear term in the expansion; then for

$$\eta \ll 1 - \left(\frac{i_{cr}}{i}\right)^2 \tag{22}$$

we have for r_i

$$r_i = \frac{\varphi}{i} = \frac{1}{i}\left\{\left[\frac{1}{(1 - i_{cr}^2/i^2)^{1/2}} - 1\right] - \frac{a_N^2 + a_S^2}{\gamma}\left[\frac{1}{(1 - i_{cr}^2/i^2)^{1/2}}\right]\right\} \tag{23}$$

from (16), (18), and (20). It is seen from the expression obtained than an increase in the transition zone (diminution in γ, see Fig. 1) in the case of large γ will result in a diminution in the real SNB sensitivity during its operation in the constant current regime.

Computations performed by means of (23) are represented in Fig. 2, from which it is seen that as i tends to i_{cr} ($i_{cr}^2/i^2 \to 1$) the sensitivity grows abruptly while the influence of the transition zone as γ increases becomes negligible and for $\gamma > 20$ and $i \leqslant 1.05 i_{cr}$, which corresponds to the real regime of SNB operation, can be neglected.

It should here be kept in mind that, strictly speaking, the analysis performed is valid only upon compliance with condition (14), i.e., for $\gamma > 5$-10. For a significant diminution in γ ($\gamma < 5$) an abrupt diminution occurs in the critical current and a reduction in the normal state zone (see Fig. 1); the bolometer practically goes over into the isothermal regime, in which connection a certain increase in the SNB sensitivity is possible in this domain of γ, where it tends in the limit $\gamma = 0$ ($T_b = T_{c1}$) to the sensitivity of the corresponding isothermal bolometer.

From the viewpoint of SNB utilization to produce an absolute thermal-radiation receiver, the analysis of SNB operation in the constant resistance regime, i.e., the realization of the thermal substitution method, is of great interest. In this case, the joint solution of (16) and (20) under the condition

$$Y_b(\eta, i) = Y_b(0, i_{cr}) \tag{24}$$

is necessary for the determination of the absorbed power W. For an exact determination of η and, therefore, of the power absorbed by the bolometer, (24) must be solved, but this can be done exactly only numerically for arbitrary γ. However, if condition (24) is considered as a function $\eta(\gamma)$ given implicitly, then by representing it as a power series in a_N^2/γ and limiting ourselves to the linear term in conformity with condition (14), we obtain

$$\eta = \frac{1}{2}\left(\frac{i_{cr}^2}{i^2} - 1\right) + \frac{a_N^2 - a_S^2}{\gamma}\frac{4a_S^2 - 1}{3 - 4a_S^2}, \tag{25}$$

from which we obtain in dimensional form by taking account of (16), (19), and (20)

$$W = R_b(i_{cr}^2 - i^2) - \frac{\rho L i^2}{A_S}\frac{4a_S^2 - 1}{\gamma}\left(a_S^2 + a_N^2 - 2\frac{a_N^2 - a_S^2}{3 - 4a_S^2}\right), \tag{26}$$

where the first term corresponds to the definition of the power absorbed by the bolometer in conformity with the substitution method, while the second describes the correction to the substitution method that is associated with the presence of a transition zone. An analysis of (26) shows that, as in the case of SNB operation in the constant current regime, the influence of the transition zone on SNB operation in the ATRR regime can be neglected for $i \approx i_{cr}$ and $\gamma > 20$; here the bolometer sensitivity under the condition $\eta = 0$ is determined by the expression

$$r_R = R_b\left(\frac{\partial i}{\partial W}\right)_R = \frac{1}{2i_{cr}}\left[1 - \frac{4a_S^2 - 1}{2\gamma}\left(a_S^2 + a_N^2 - 2\,\frac{a_N^2 - a_S^2}{3 - 4a_S^2}\right)\right] \qquad (27)$$

and grows abruptly with the diminution in the critical current. As follows from (18), the diminution of i_{cr}, meaning the increase in r_R also, for a given geometry L can be achieved both by an increase in the specific resistivity ρ/A_S, i.e., by the deposition of thinner films, and also by the diminution of the temperature difference $\Delta T = T_{c1} - T_b$, i.e., by raising the bolometer operating temperature and therefore diminishing γ. However, it should here be kept in mind that a significant diminution in γ already results in a diminution in the bolometer sensitivity because of the magnification of the influence of the transition zone. Moreover, for $\gamma < 20$ the corrections related to the influence of the transition zone must also be taken into account in (26) for an exact realization of the substition method. For γ less than 5-10, these corrections become so significant in connection with the abrupt reduction of the normal state zone that the substitution method is not realized in practice, and utilization of the SNB as an ATRR can result in large errors.

In practice this means that preference should be given to superconductors of the first kind (In, Sn, Pb, and their alloys) for the production of ATRR on the basis of SNB, since the difference in the critical temperatures $\Delta T_c \ll 0.01°K$ and the bolometer supercooling by just 0.1°K will permit obtaining high sensitivity ($r_R = 10^2$-10^3 V/W) for $\gamma \gg 10$. At the same time utilization of superconductors of the second kind (Nb and its alloys) for this purpose, wherein ΔT_c can reach 1-2°K, requires bolometer supercooling of 10°K and more, which will result in a diminution in the sensitivity because of the increase in i_{cr}; here $\gamma \ll 10$, which reduces the measurement accuracy.

It is therefore seen that the transition torsion parameter in the temperature can affect the SNB operating regime in a substantial manner. For a reasonable selection of this parameter, the production of an absolute thermal-radiation receiver on the basis of a SNB is possible with a sufficiently high sensitivity, independently of the nature of the incident thermal flux distribution.

NOTATION

λ_e, effective heat-conduction coefficient of the bolometer; ρ, specific resistivity; A_s, film cross-sectional area; A_0, substrate cross-sectional area; $y = x/L$, dimensionless distance from the center of the bolometer; L, distance between the center and the base of the bolometer; T_{c1}, upper critical transition temperature bound; T_{c2}, lower critical transition temperature bound; T_b, temperature of the bolometer base; $\gamma = (T_{c1}-T_b)/(T_{c1}-T_{c2})$, transition torsion parameter in the temperature; $\Theta = (T-T_{c1})/(T_{c1}-T_b)$, dimensionless temperature; W, power absorbed by the bolometer; R_b, total bolometer resistance; r, sensitivity; i_{cr}, critical current.

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